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IMPROVED MOVING-TARGET-INDICATOR
FILTERING FOR PHASED ARRAY RADARS

Robert H. Fletcher, Jr., et al

Army Missile Command
Redstone Arsenal, Alabama

28 March 1973

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A technique is described for using recursive digital filters in the moving target indicator of phased array radars. This technique, called initialization, is effective in reducing the severe transient effects normally exhibited by these filters for large clutter input signals. Consequently, the superior frequency characteristic of recursive filters can be realized. The problem is addressed from the state variable point of view and a general initialization expression is derived. This procedure is illustrated for example filters, and typical moving target indicator output sequences are shown. The concept of "dynamic" frequency response is introduced and used to demonstrate the effectiveness of initialization for moving target indicator input sequences of varying length.

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1. Introduction

Many radars presently employ a digital implementation of the delay-line canceler for the detection of moving targets in a background of clutter. This moving target indicator (MTI), which is actually a nonrecursive digital filter, may be reasonably effective against low frequency components of clutter. The standard cancelers, however, exhibit rather poor pass-band frequency characteristics, and a number of techniques have been developed to improve their response by using different feed-forward multiplier coefficients [1]. These nonrecursively implemented filters, which possess finite-duration impulse responses, are somewhat limited in the amount of frequency response shaping that can be obtained. Recursively implemented filters, on the other hand, offer superior frequency characteristics over nonrecursive filters of comparable hardware [2]. Due to the feedback nature of these filters, however, the transient response due to an impulse input is much more severe. In fact these recursively implemented filters are frequently called infinite-duration impulse response filters.

2. Transient Problem for Recursive MTI Filters

Because of the relatively poor transient performance of recursive filters, these filters have seldom been employed for MTI purposes. The transient response problem is particularly severe in a strong ground clutter environment. For this case the clutter returns may be orders of magnitude larger than those of a target and effectively appear as a large step input to the filter. This can produce such excessive ringing in the filter output that the target will be masked until the transient response has settled. Furthermore, the oscillatory characteristic of the MTI increases in magnitude as the bandwidth of the filter increases, i.e., as the clutter notch at zero doppler frequency is narrowed. This effect is graphically depicted in Figure 1 which shows the unit step response for three four-pole Butterworth filters. As the 3-dB cutoff frequency is lowered, the transient response of these filters becomes increasingly severe. Since most radars are limited in the time allocated to each sector of their scan, the long settling times required reduce the effectiveness of the recursive implementation.

For conventional scanning radars the modulation due to the scanning antenna pattern reduces the severity of the transient response from large sharply defined clutter sources. Nevertheless the transient problem may still be significant, and a trade-off between frequency response characteristics and acceptable transient behavior is usually required. While this has resulted in some satisfactory recursive MTI designs, the nonrecursive implementation has generally been employed. However, for radars that use step scanning such as those with phased array antennas, a new technique [3] has been suggested to minimize the poor transient performance of recursive MTI filters.

UNIT STEP RESPONSES FOR FOUR-POLE BUTTERWORTH MTI FILTERS

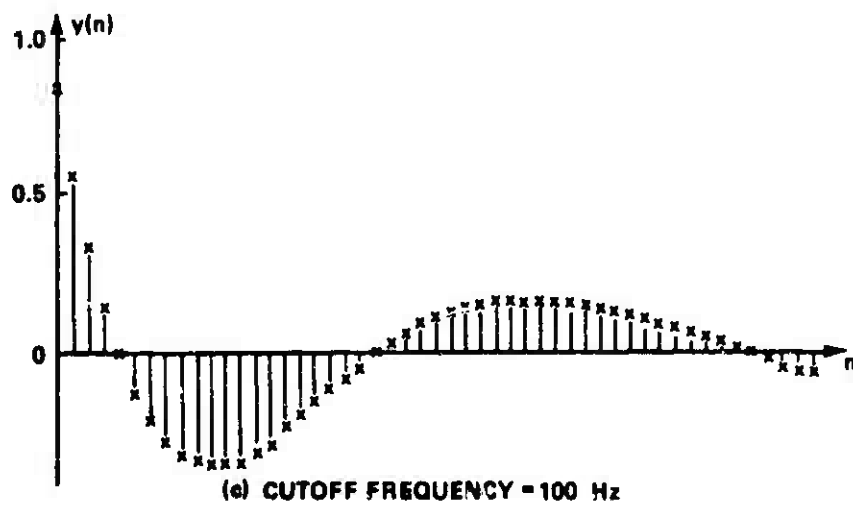
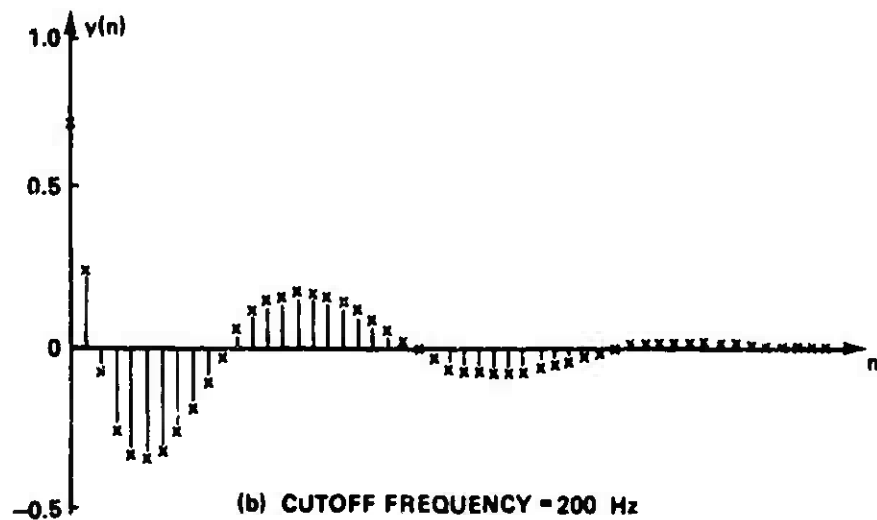
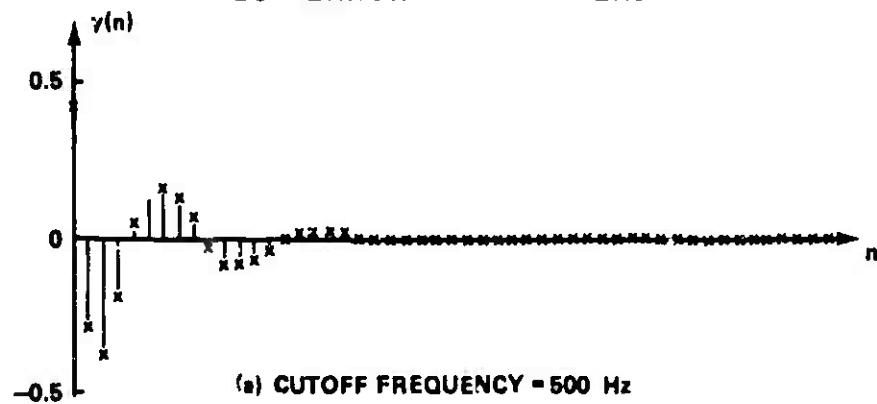


Figure 1. Unit Step Responses for Four-Pole Butterworth MTI Filters

3. Initialization Technique

This technique, which is called initialization, takes advantage of the discrete nature of phased array scanning to reduce transient effects. In contrast with conventional scanning systems which process returns continuously, phased array systems reinitiate the processing each time the beam is stepped to a new position. Hence, the initial return from each new beam position can be used to apply initial conditions to the MTI for processing the remaining returns from that position. Since ground clutter usually exhibits relatively low frequency components, the clutter returns can be approximated by a step input equal in magnitude to the first return for each beam position. The steady-state values that would normally appear in the filter memory elements after an infinitely long sequence of these inputs can therefore be immediately calculated and loaded into the filter to suppress the transient response.

The initialization technique can be derived by considering the general state variable block diagram of Figure 2. This linear, time-invariant single input-single output system represents the digital MTI with the state variables $\underline{x}(n)$ defined as the outputs of the MTI filter registers.*

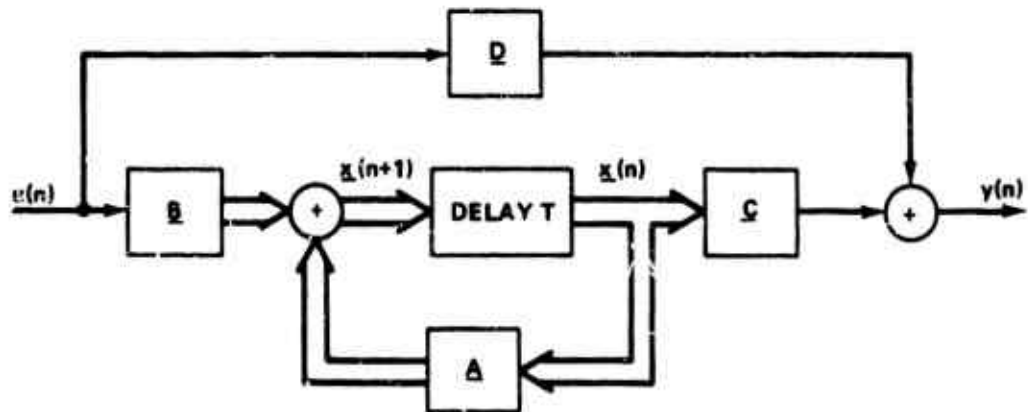


Figure 2. General State Variable Representation of Digital MTI

The corresponding state variable equations are [4]

$$\underline{x}(n+1) = \underline{A}\underline{x}(n) + \underline{B}u(n) \quad (1)$$

$$y(n) = \underline{C}\underline{x}(n) + \underline{D}u(n) \quad (2)$$

*The abbreviated notation $\underline{x}(n) \triangleq \underline{x}(nT)$ is used in this work.

and the solution to the state equation is

$$\underline{x}(n) = \underline{A}^n \underline{x}(0) + \sum_{m=0}^{n-1} \underline{A}^{n-1-m} \underline{B}u(m) \quad . \quad (3)$$

For the initialization technique under consideration, the basic procedure is to immediately force the registers of the filter to their steady-state values by applying initial conditions to the filter. Therefore, the required initial conditions, $\underline{x}(0)$, must be such that for a step input the states remain constant, specifically,

$$\underline{x}(n+1) = \underline{x}(n) = \underline{x}(0), \quad \text{for all } n > 0 \quad . \quad (4)$$

It then follows from (3) that

$$\begin{aligned} \underline{x}(n+1) - \underline{x}(n) &= (\underline{A}^{n+1} - \underline{A}^n) \underline{x}(0) + \underline{A}^n \underline{B}u(0) \\ &+ \sum_{m=0}^{n-1} \underline{A}^{n-1-m} \underline{B}u(m+1) - \sum_{m=0}^{n-1} \underline{A}^{n-1-m} \underline{B}u(m) \quad . \quad (5) \end{aligned}$$

Substituting $\underline{x}(n+1) - \underline{x}(n) = 0$ from (4) and $u(m+1) - u(m) = 0$ from the step input assumption yields

$$\underline{0} = \underline{A}^n (\underline{A} - \underline{I}) \underline{x}(0) + \underline{A}^n \underline{B}u(0) \quad . \quad (6)$$

Consequently, the initialization expression can be written as

$$\underline{x}(0) = (\underline{I} - \underline{A})^{-1} \underline{B}u(0) \quad (7)$$

by excluding the trivial case $\underline{A} = \underline{0}$ and assuming that the inverse in (7) exists. This expression gives the values, $\underline{x}(0)$, that must be stored in the MTI memory registers once the initial return from each new beam position, $u(0)$, is known.

Although the use of (7) will result in the suppression of the transient response to a step input, it does not ensure that the MTI output will be at a zero level in its steady-state condition. This suggests that a restriction should be placed on the two coupling

matrices \underline{C} and \underline{D} of the output equation (2) to ensure that the steady-state output to a step input is zero. The restriction can be obtained by substitution of (3) into (2) giving an expression for the MTI output as a function of the input and initial states, i.e.,

$$y(n) = \underline{C} \left[\underline{A}^n \underline{x}(0) + \sum_{m=0}^{n-1} \underline{A}^{n-1-m} \underline{B} u(m) \right] + \underline{D} u(n) \quad (8)$$

Now, by using the initialization expression of (7), the output can be written as

$$y(n) = \underline{C} \underline{A}^n (\underline{I} - \underline{A})^{-1} \underline{B} u(0) + \underline{C} \sum_{m=0}^{n-1} \underline{A}^{n-1-m} \underline{B} u(m) + \underline{D} u(n) \quad (9)$$

For a step input it is desired that the output be zero for all n and in particular for $n = 0$. Consequently, (9) reduces to

$$0 = \underline{C} \underline{A}^0 (\underline{I} - \underline{A})^{-1} \underline{B} u(0) + \underline{D} u(0) \quad (10)$$

and it follows that the desired relationship is

$$\underline{D} = -\underline{C} (\underline{I} - \underline{A})^{-1} \underline{B} \quad (11)$$

4. Example MTI Filters

The application of (7) and (11) can be illustrated by considering the MTI filter of Figure 3(a). By inspection of this block diagram, the state variable equations are

$$\begin{bmatrix} x_1(n+1) \\ x_2(n+1) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ b_2 & b_1 \end{bmatrix} \begin{bmatrix} x_1(n) \\ x_2(n) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(n) \quad (12)$$

and

$$y(n) = \begin{bmatrix} 1 + b_2 & b_1 - 2 \end{bmatrix} \begin{bmatrix} x_1(n) \\ x_2(n) \end{bmatrix} + [1] u(n) \quad (13)$$

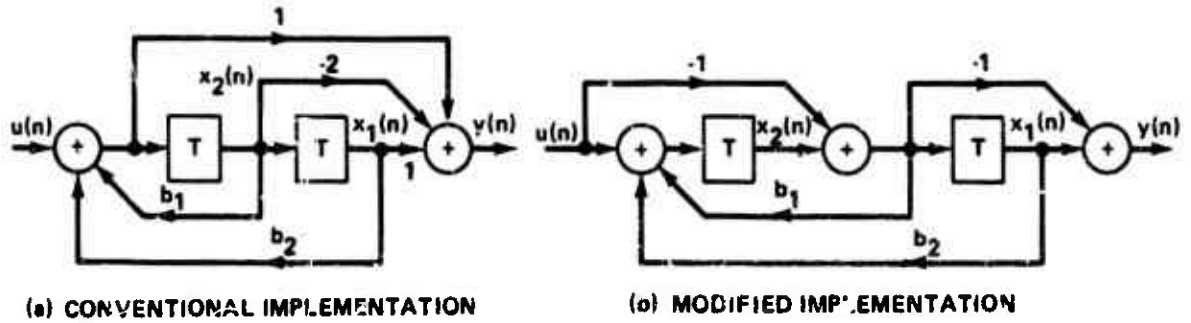


Figure 3. Recursive MTI Filters

It then follows from (7) that the initialization values to be used are

$$\begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \frac{1}{1 - b_1 - b_2} \begin{bmatrix} 1 - b_1 & 1 \\ b_2 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad u(0) = \frac{u(0)}{1 - b_1 - b_2} \begin{bmatrix} 1 \\ 1 \end{bmatrix} \quad (14)$$

and from (11) the required \underline{D} is

$$\underline{D} = - \frac{1}{1 - b_1 - b_2} \begin{bmatrix} 1 + b_2 & b_1 - 2 \end{bmatrix} \begin{bmatrix} 1 - b_1 & 1 \\ b_2 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = [1] \quad (15)$$

which is satisfied for Figure 3(a). Consequently, the first return from each beam position is multiplied by $\{1/(1 - b_1 - b_2)\}$, this product is loaded in the filter registers, and the remaining returns from that beam position are processed in the normal manner. However, (14) illustrates a problem with the conventional implementation scheme of Figure 3(a). As the filter cutoff frequency approaches zero, the denominator of the coefficient term in (14) also approaches zero. This results in a very large magnification of the input signal and a corresponding increase in the number of bits required to represent the register values. An improved implementation [5] having the same steady-state frequency characteristic as the conventional implementation but different state variable equations is shown in Figure 3(b). Since the state equations are

$$\begin{bmatrix} x_1(n+1) \\ x_2(n+1) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ b_2 & b_1 \end{bmatrix} \begin{bmatrix} x_1(n) \\ x_2(n) \end{bmatrix} + \begin{bmatrix} -1 \\ 1 - b_1 \end{bmatrix} u(n) \quad (16)$$

and

$$y(n) = \begin{bmatrix} 1 & -1 \end{bmatrix} \begin{bmatrix} x_1(n) \\ x_2(n) \end{bmatrix} + [1]u(n) \quad , \quad (17)$$

the initialization values are

$$\begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \frac{1}{1 - b_1 - b_2} \begin{bmatrix} 1 - b_1 & 1 \\ b_2 & 1 \end{bmatrix} \begin{bmatrix} -1 \\ 1 - b_1 \end{bmatrix} u(0) = \begin{bmatrix} 0 \\ u(0) \end{bmatrix} \quad (18)$$

and the required \underline{D} is

$$\underline{D} = \frac{-1}{1 - b_1 - b_2} \begin{bmatrix} 1 & -1 \end{bmatrix} \begin{bmatrix} 1 - b_1 & 1 \\ b_2 & 1 \end{bmatrix} \begin{bmatrix} -1 \\ 1 - b_1 \end{bmatrix} = [1] \quad . \quad (19)$$

Expression (18) indicates a particularly convenient form of initialization in which the first memory register of the filter is simply loaded with the initial radar return while the second register is zeroed. The registers in any additional cascaded sections following the first are also zeroed. Moreover, the dynamic range requirements for the registers of the modified implementation are reduced in comparison with those of the conventional implementation since the multiplier coefficient in (18) is now unity. Both of these implementations satisfy the relationship of (11) which ensures that the steady-state output will be zero for a step input. Any normal MTI filter with a clutter notch of zero gain at zero frequency will satisfy this relationship.

5. Four-Pole Filter

The filters of the preceding section may be cascaded as shown in Figure 4 to give a very useful filter configuration. As previously mentioned, the initialization of this filter consists of loading $u(0)$ into the first register, $x_4(n)$ in Figure 4, and then zeroing the remaining registers. This can readily be verified from the state and output equations for this filter, (20) and (21).

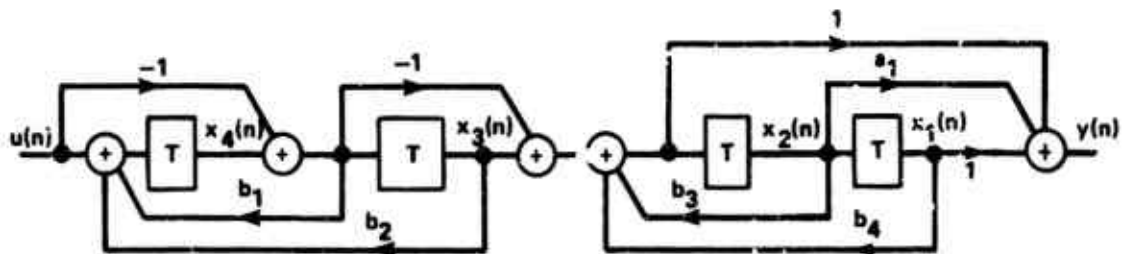


Figure 4. Four-Pole Recursive Filter

$$\begin{bmatrix} x_1(n+1) \\ x_2(n+1) \\ x_3(n+1) \\ x_4(n+1) \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ b_4 & b_3 & 1 & -1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & b_2 & b_1 \end{bmatrix} \begin{bmatrix} x_1(n) \\ x_2(n) \\ x_3(n) \\ x_4(n) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ -1 \\ 1 - b_1 \end{bmatrix} u(n) \quad (20)$$

and

$$y(n) = \begin{bmatrix} 1 + b_4 & a_1 + b_3 & 1 & -1 \end{bmatrix} \begin{bmatrix} x_1(n) \\ x_2(n) \\ x_3(n) \\ x_4(n) \end{bmatrix} + [1]u(n) \quad (21)$$

Using (7) and A and B from (20), the initialization values are seen to be

$$\begin{bmatrix} x_1(0) \\ x_2(0) \\ x_3(0) \\ x_4(0) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ u(0) \end{bmatrix}, \quad (22)$$

which verifies the preceding comments.

Another important feature of this filter is that (11) is satisfied regardless of the value of a_1 . Therefore, the first filter section assures a zero dc gain of the filter, and the a_1 of the second section can be used to provide a notch in the frequency response at some frequency other than zero.

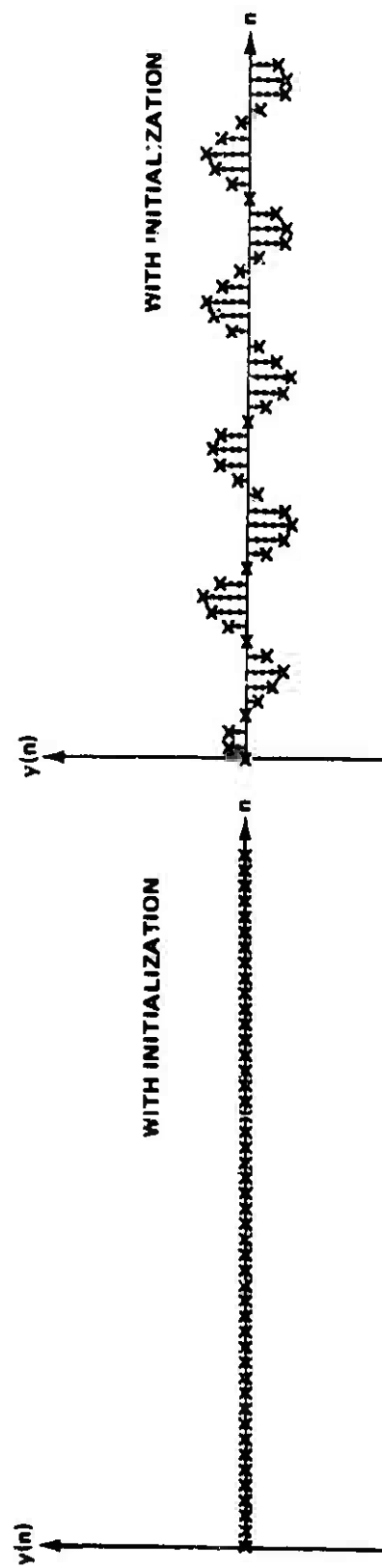
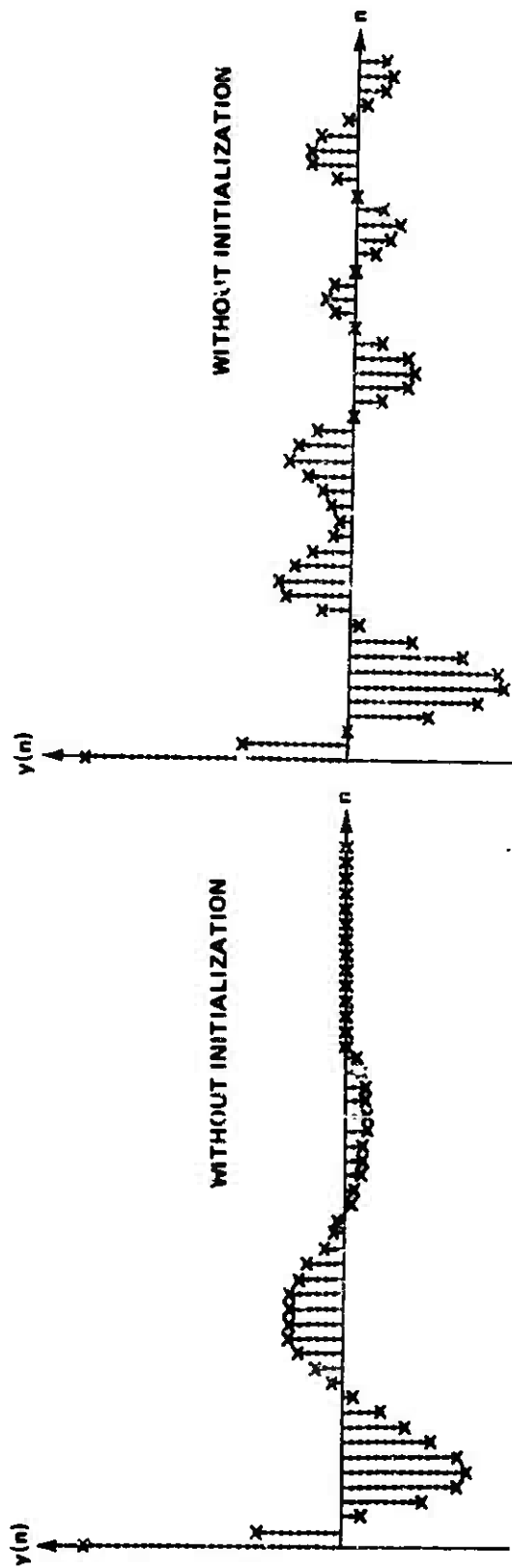
6. Transient Response Comparisons

It has been assumed for the purposes of initialization that the first return is due only to clutter, but obviously this is not always the case. If a target signal is present along with the clutter, there may be an error in the initialization value which will produce a transient effect in the filter output. A similar situation exists when there is a target signal alone. Generally, these effects are small when compared with the ringing caused by clutter in a recursive MTI with no initialization.

The improvement in typical MTI transient responses is illustrated in Figure 5 for systems with and without initialization. These responses are for a four-pole Butterworth filter having a 3-dB cutoff frequency of 4 percent of the radar pulse-repetition frequency (PRF). The unit step response of this filter was given in Figure 1(b). The power spectral density of the simulated ground clutter return consisted of a zero-frequency component and a Gaussian fluctuating component with an rms frequency spread equal to 0.5 percent of the PRF and ratio of stationary-to-fluctuating clutter power of 0.8 [6, 7]. The plots of Figure 5(a) are the MTI responses for a clutter return only while Figure 5(b) shows the filter output when the input signal consists of clutter plus a target. The target return had a doppler frequency equal to 10 percent of the PRF and the clutter-to-target power ratio was 25 dB. It is apparent from the responses without initialization that the transient effect masks the desired target signal for an appreciable period of time. With initialization, on the other hand, this effect is significantly reduced and the target can be immediately detected.

7. Dynamic Frequency Response

The effectiveness of the initialization technique for other doppler frequencies was investigated by simulating a radar quadrature processor and obtaining a type of frequency response which incorporates the transient effects. The processor input signal consisted of the in-phase (I) and quadrature (Q) components of a complex signal $\exp(j2\pi f_d T)$. For each doppler frequency, f_d , chosen in the range 0 to $\text{PRF}/2$, input sequences of N pulses were generated to represent these I and Q signals. These sequences were processed by the respective MTI's and then



(a) CLUTTER RETURN ONLY (b) CLUTTER AND TARGET RETURN

Figure 5. Comparison of MTI Transient Responses

recombined by taking the square root of the sum of the squares of the MTI outputs. The N recombined values were then integrated to give a processor output residue $S(f_d)$. This process is illustrated in Figure 6.

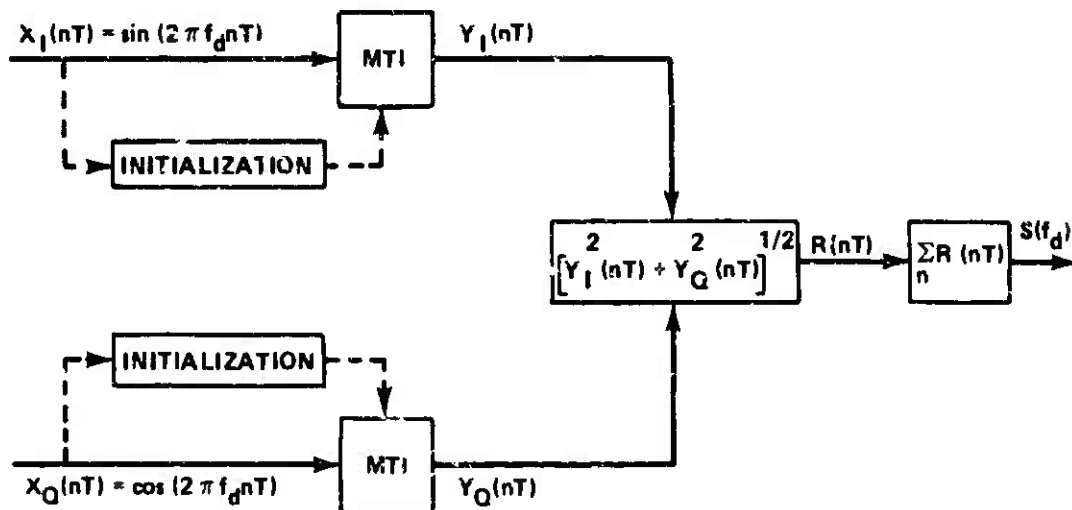


Figure 6. Radar Quadrature Processor

Figures 7 through 9 show plots of the normalized $S(f_d)$ values both with and without initialization for pulse sequences of varying lengths. These figures, called "dynamic" frequency responses [3], are for the three Butterworth filters whose step responses are given in Figure 1. These curves clearly indicate the improved performance offered by the initialization technique. It is also apparent from these curves that the "dynamic" frequency response characteristic approaches the steady-state MTI response as the number of pulses processed increases.

8. Conclusions

An initialization technique has been described which provides significant improvement in the MTI performance of phased array radars. Moreover, this improved performance is obtained with virtually no increase in hardware over a conventional recursively implemented filter. It has been demonstrated that the improvement is a function of the number of pulses per beam position and is degraded as the number of pulses is decreased. However, for as few as six pulses per position, the initialized recursive filter appears to have an adequate characteristic. It should be emphasized that these curves were obtained using the coefficients for a standard Butterworth filter which is optimum only for steady-state performance. A further improvement should be obtained if MTI multiplier coefficients are used that optimize the processor dynamic response for a given number of pulses.

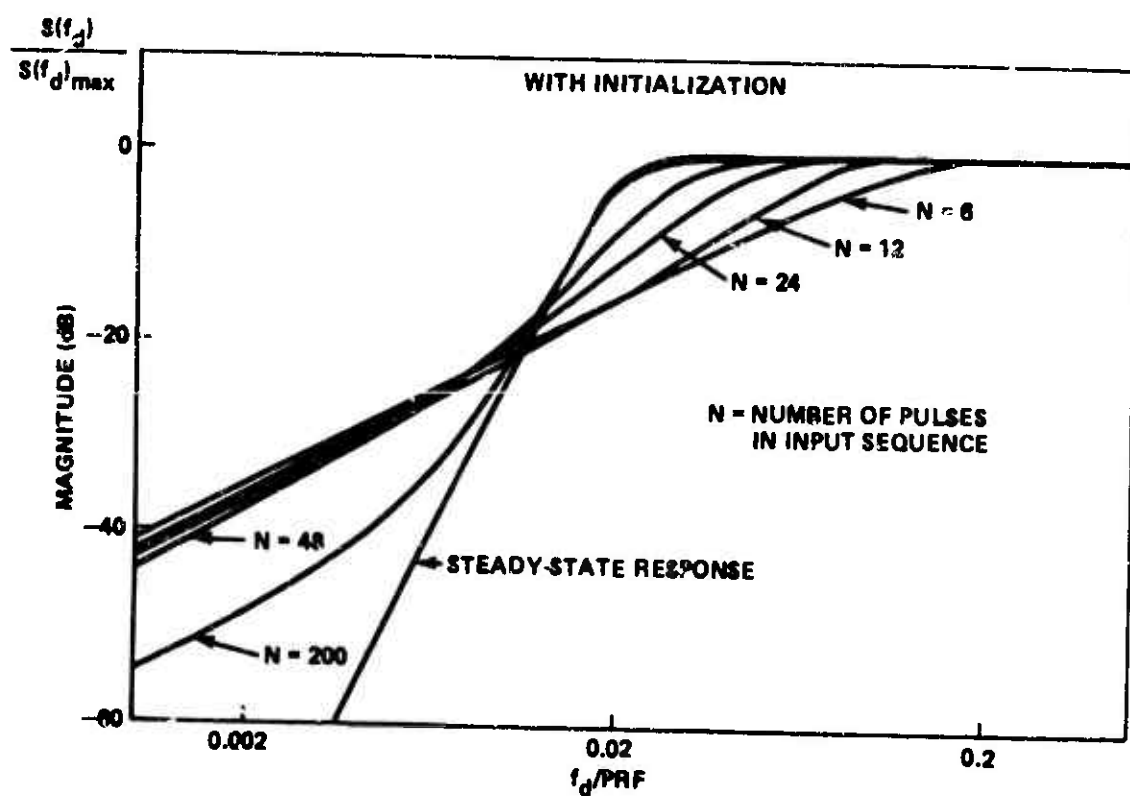
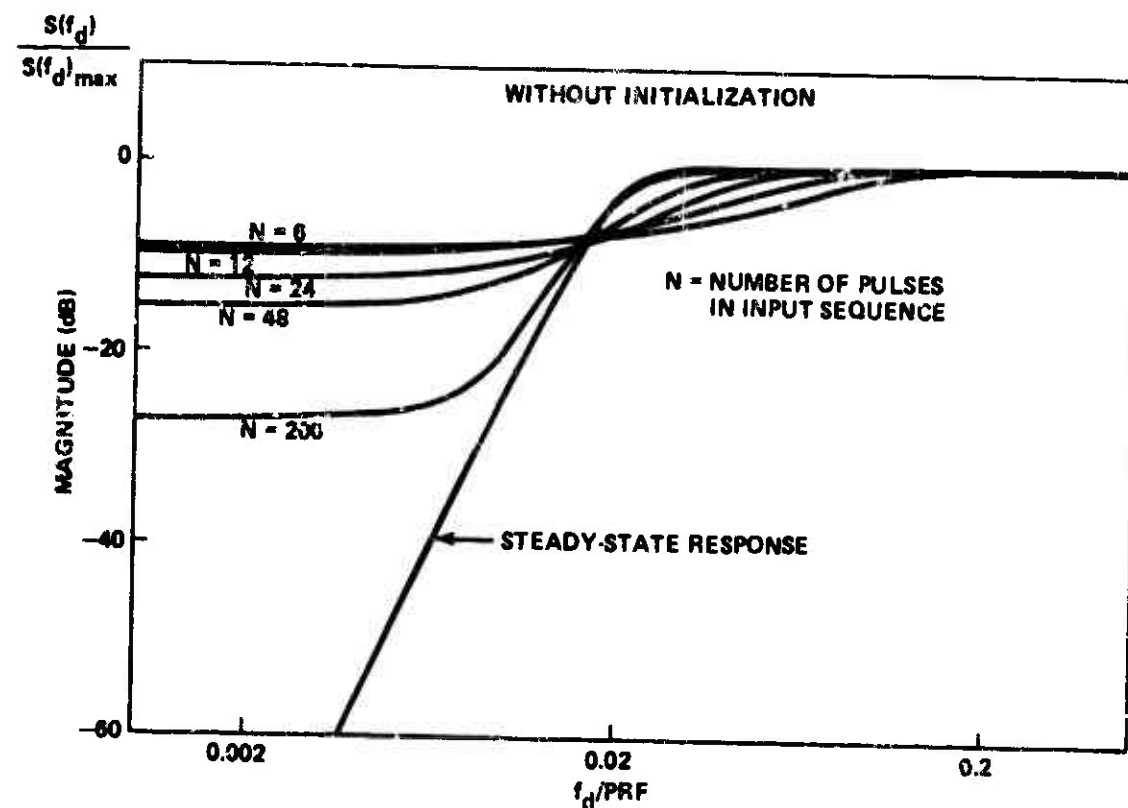


Figure 7. Dynamic Frequency Responses for Filter Cutoff Frequency of 0.02 PRF

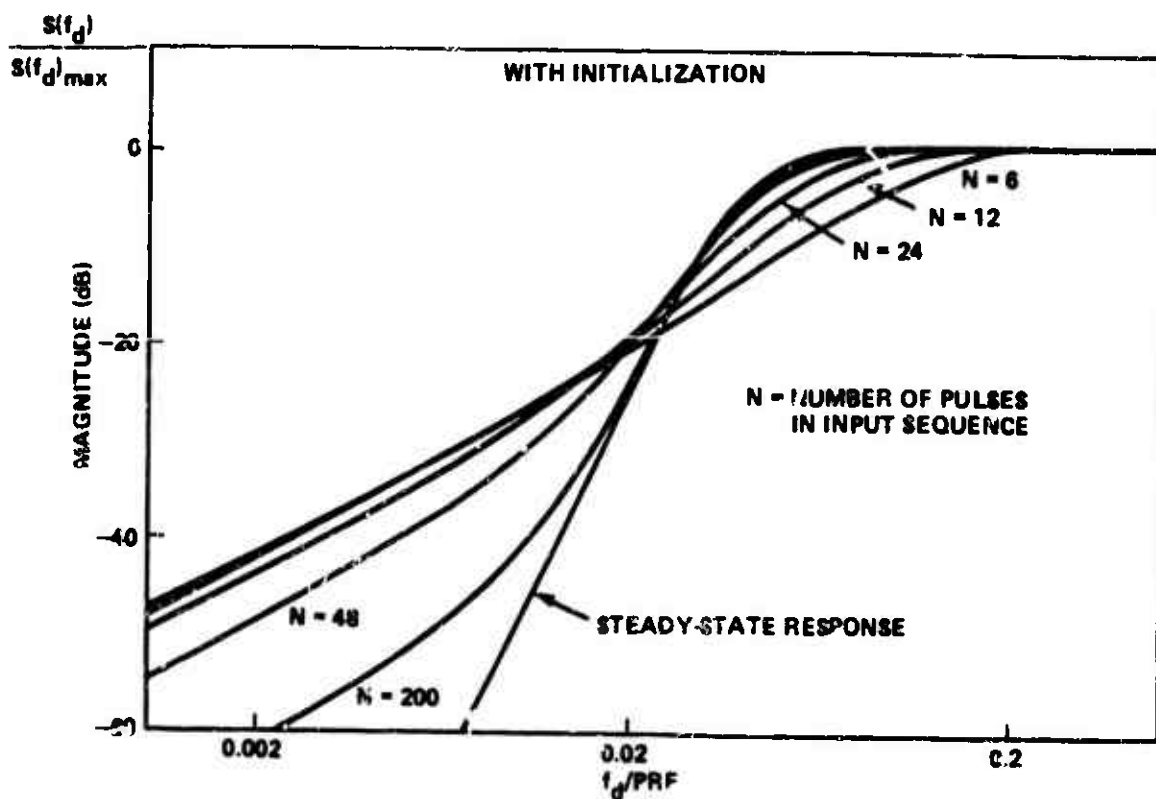
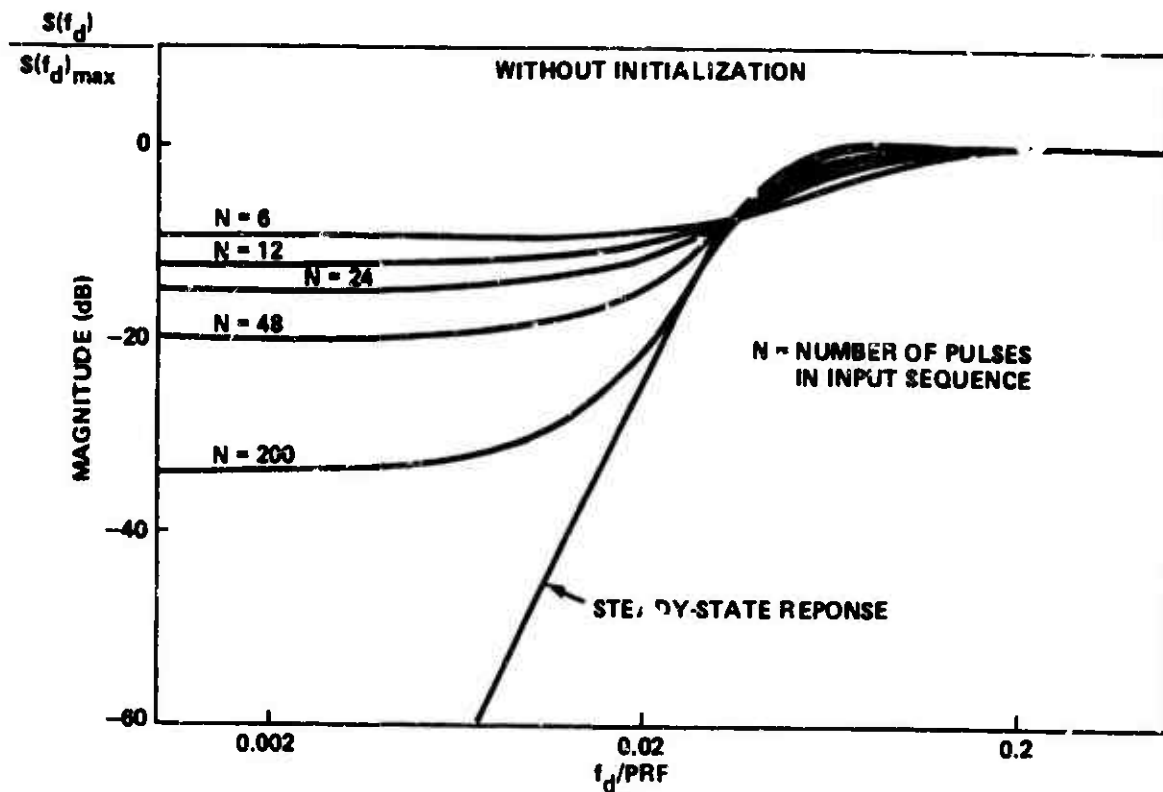


Figure 8. Dynamic Frequency Responses for Filter Cutoff Frequency of 0.04 PRF

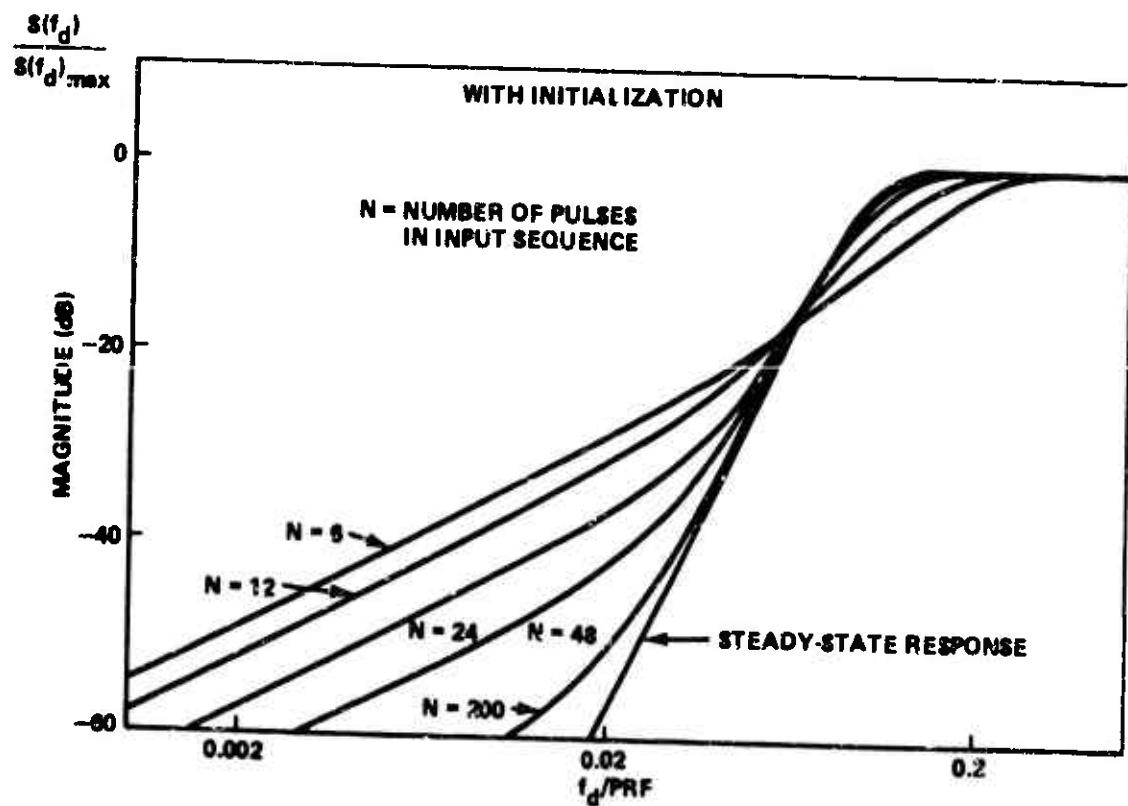
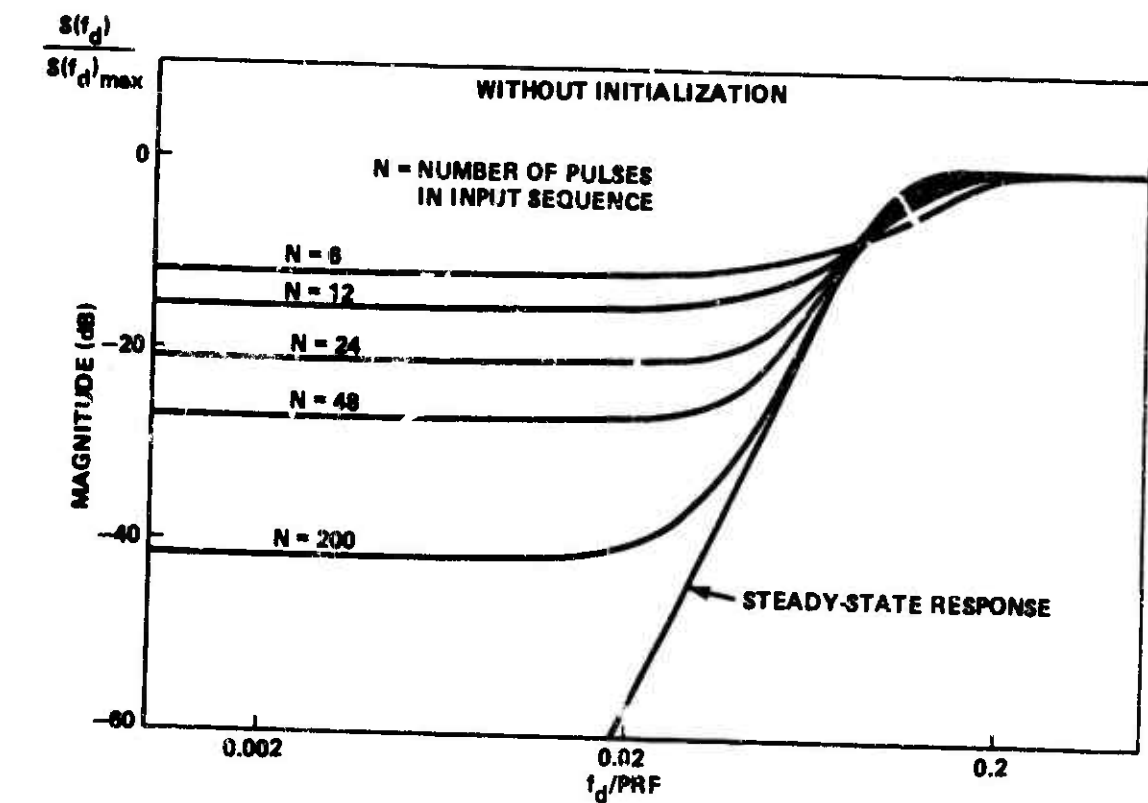


Figure 9. Dynamic Frequency Responses for Filter Cutoff Frequency of 0.1 PRF

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